Spatial Interpolation & Geostatistics

Kriging – Step 1

- Describe spatial variation with Semivariogram

Kriging – Step 2

- Summarize spatial variation with a function
  - Several choices possible; curve fitting defines different types of Kriging (circular, spherical, exponential, gaussian, etc.)

Kriging – Step 2

- Key features of fitted variogram:
  - Nugget: semivariance at \( d = 0 \)
  - Range: \( d \) at which semivariance is constant
  - Sill: constant semivariance beyond the range

Distance between pairs of points

Lag Mean

Distance between pairs of points

Lag

Distance between pairs of points

Lag

Semivariance

Distance (d)

Nugget

Sill

Range

“Point cloud”

Map

\[ \frac{Z_i - Z_j}{2} \]
Kriging – Part II

- **Goal**: predict values where no data have been collected
- Relies on first establishing:
  - **DEPENDENCY** – z is, in fact, correlated with distance, i.e. Tobler’s Law
  - **STATIONARITY** – z values are stochastic (except for spatial dependency they are randomly distributed) and have no other dependence – use “detrending” or transformation tools if not Gaussian
  - **DISTRIBUTION** – works best if data are Gaussian. If not, they have to first be made close to Gaussian.

ESRI Geostatistical Analyst Products

- **Map types**:
  - Prediction – contours of interpolated values
  - Prediction Standard Errors – show error distribution, as quantified by minimized RMS error (see below)
  - Probability – show probability of values exceeding a specified threshold
  - Quantile – show where thresholds overestimate or underestimated predictions

Some Kriging Products

- Prediction map – interpolated values – like produced by other interpolation techniques
- Probability map - showing where critical values exceeded
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Spatial Dependence:
Semivariogram and Semivariogram Surface

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2. Stationarity

- Test with semivariogram & cross-validation plots

Violates Stationarity; detrend first

Covariance Surface, Cool!

Stationarity is the assumption that the covariance between two points is the same for a given distance and direction, regardless of which two points are chosen.

2. STATIONARITY - Randomness

- Data variance and mean is the same at all localities (or within a neighborhood of nearest points); data variance is constant in the neighborhood of investigation
- Correlation (covariance) depends only on the vector that separates localities, not exact locations, number of measurement or direction (anisotropy)

California Ozone Demo. – next time?

- Data in “Geostat_demo” zip folder in Canvas Module for Week 10

ArcGIS Kriging Processing Steps

1. Add and display the data
2. Explore the data’s statistical properties
3. Select a model to create a surface – make a prediction map!
4. Assess the result
5. Compare to other models
Data Exploration

1. Examine the distribution – normal (Gaussian)? Transformation to normal required?
   - Histograms and QQ Plots

2. Identify trends, if any
   - Trend Analysis

3. Understand spatial autocorrelation and directional influences
   - Semivariogram analysis

Data Exploration:
Examine the Distribution

- Normal (Gaussian) distribution? (central value, spread, symmetry; mean and median the same?) Standard Normal: mean = 0, standard dev. = 1
  - Transformation to normal required?
  - Histogram tool, QQPlot tool (compare real and standard normal distributions)

Data Exploration:
Identify Trends, If Any

- Underlying trends affect Kriging assumption of randomness – remove and work with “residuals”
- Trend Analysis tool

Data Exploration:
Spatial Autocorrelation & Directional Influences

- Variogram Analysis:
  - Look for correlation with distance
  - Look for directional trends among pairs of points
  - Semivariogram/Covariance Cloud tool

Exhibits Spatial Autocorrelation

Directional Influence shown here – pairs in NW-SE direction show largest covariance over shortest distances

- nearly Normally Distributed data
Mapping Ozone Concentration

1. Incorporate results of Data Exploration into Model selection
   - This example:
     - Remove underlying trends discovered during data exploration that have a rational explanation. (Analysis is then performed on residuals and trend surface is added back into final surface) = "Detrending"
     - Remove directional trends between pairs of points – in certain directions closer points are more alike than in other directions = "anisotropy removal"

2. Define search neighborhood for interpolation (c.f. I.D.W.)
   - Use a search ellipse (or circle) to find nearest neighbors; specify radii of ellipse, min. & max. number of points per sectors

3. Examine Cross Validation plot
   - Predicted vs. Measured for subset(s) of the data
     - "Mean error" should be close to zero
     - "RMS error" and "mean standardized error" should be small
     - "RMS standardized error" should be close to one.

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Comparing Model Results

- Cross validation comparisons:
  - "Mean error" should be close to zero
  - "RMS error" and "mean standardized error" should be small
  - "RMS standardized error" should be close to one.

Probability Mapping with Indicator Kriging

- Task: Make a map that show the probability of exceeding a critical threshold, e.g. 0.12 ppm ozone for an 8 hr. period
- Technique:
  - Transform data to a series of 0s and 1s according to whether they are above or below the threshold
  - Use a semivariogram on transformed data; interpret indicator prediction values as probabilities