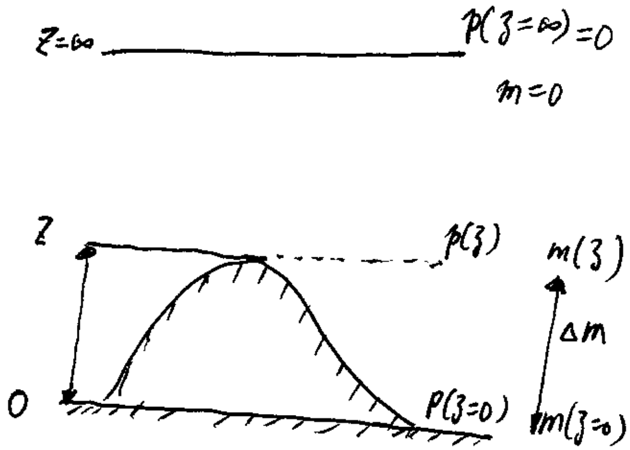
Physical Climatology Homework #1

1. As shown in the right, the fraction of mass of the atmosphere below z is defined as



$$\frac{\Delta m}{m(z=0)} = \frac{m(z=0) - m(z)}{m(z=0)}$$

$$= \frac{p(z=0)/g - p(z)/g}{p(z=0)/g}$$ (following eqn 1.8 in the Hartmann book)

$$\doteq \frac{p(z=0) - p(z=0)\exp(-\frac{z}{H})}{p(z=0)}$$

$$= 1 - \exp(-\frac{z}{H})$$

$$= 1 - \exp(-\frac{8848}{7600})$$

$$= 1 - 0.3122$$

$$= 0.6878$$

$$\doteq 69\%$$

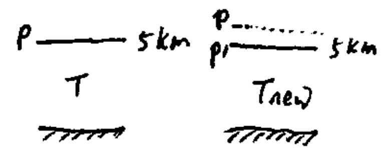
2. The mean temperature (T) for the scale height H is: $T = \frac{gH}{R} = \frac{9.80665 \times 7600}{287}$
$= 259.69$ K

If the mean temperature is warmed by 5°C, i.e., T increased by 5 K

$T_{new} = T + 5 = 259.69 + 5 = 264.69$ K, $H_{new} = \frac{R T_{new}}{g} = \frac{287 \times 264.69}{9.80665} \doteq 7746$ m

The pressure change at 5 km is

$$\Delta P = P' - P = P(z=0)\left[\exp(-\frac{5000}{7746}) - \exp(-\frac{5000}{7600})\right]$$

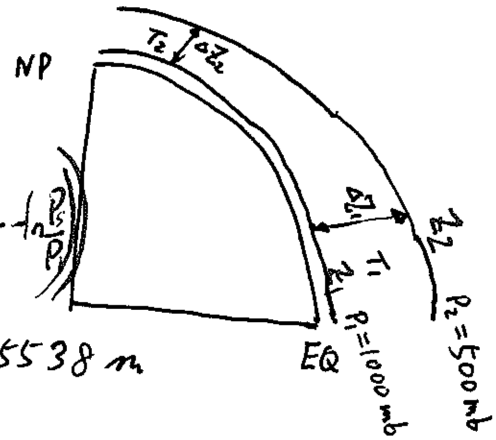


$\doteq 6.48 \times 10^{-3} \times 101325$

$\doteq 657$ Pa

It is increased by 657 Pa.

3. The 1000 mb—500 mb thickness is



$$\Delta Z = \frac{R\bar{T}}{g} \ln \frac{P_1}{P_2}$$

$$\left(\because P = P_s e^{-Z/H} \quad \therefore Z = H \ln \frac{P_s}{P} \quad Z_2 - Z_1 = H \left(\ln \frac{P_s}{P_2} - \ln \frac{P_s}{P_1} \right) \right)$$

$$\bar{T}_1 = 273\,K, \quad \Delta Z_1 = \frac{287 \times 273}{9.80665} \times \ln \frac{1000}{500} = 5538\ m$$

$$\bar{T}_2 = 250\,K, \quad \Delta Z_2 = \frac{287 \times 250}{9.80665} \ln \frac{1000}{500} = 5071\ m$$

The 1000-500 mb thickness is greater in warmer latitudes.

4. a) From $\frac{\partial p}{\partial z} = -\rho g = -\frac{pg}{RT}$ (*)

Using $T = T_0 - \Gamma z$ in (*): $\frac{dp}{p} = -\frac{g}{R} \frac{dz}{T_0 - \Gamma z}$

Integrating both sides from $(z=0, p=p_0)$ to (z, p)

$$\ln \frac{p}{p_0} = \frac{g}{R\Gamma} \ln \frac{T_0 - \Gamma z}{T_0} \qquad \left(\frac{p}{p_0}\right)^{\frac{R\Gamma}{g}} = 1 - \frac{\Gamma}{T_0} z$$

$$\therefore z = \frac{T_0}{\Gamma}\left[1 - \left(\frac{p}{p_0}\right)^{\frac{R\Gamma}{g}}\right] \qquad (**)$$

b) $T_0 = 288K$, $P_0 = 1013.25$ hPa, $P = 850$ mb $= 850$ hPa

$R = 287\ JKg^{-1}K^{-1}$, $\Gamma = 6.5\ °C \cdot km^{-1} = 6.5\ K \cdot km^{-1} = 0.0065\ K\,m^{-1}$

$$z = \frac{288}{0.0065} \times \left[1 - \left(\frac{850}{1013.25}\right)^{\frac{287 \times 0.0065}{9.80665}}\right] = 1456.28\ m$$

c) $P_0 = 1078$ hPa, $P = 850$ mb $= 850$ hPa, $R = 287\ JKg^{-1}K^{-1}$, $\Gamma = 0.0065\ K\,m^{-1}$

$T_0 = 217\,K$

Using (**): $z = \frac{T_0}{\Gamma}\left[1 - \left(\frac{P}{P_0}\right)^{\frac{R\Gamma}{g}}\right] = \frac{217}{0.0065}\left[1 - \left(\frac{850}{1078}\right)^{\frac{287 \times 0.0065}{9.80665}}\right] = 1475.49\,m$

using $z' = \frac{RT_0}{g} \ln \frac{P_0}{P}$ because the lower atmosphere is isothermal

$$= \frac{287 \times 217}{9.80665} \times \ln \frac{1078}{850} = 1509.09\ m$$

$\therefore$ Using (**) can give an error

$$\Delta = z - z' = -33.60\ m$$

Using (b) gives an error $\Delta = z - z' = 1456.28 - 1509.09$

$= -52.81\ m$