Physical Climatology Problem Set # 4

Hartmann Book #114,

(2). Derive (4.18) from (4.15) and (4.17).

(4.18) $\tau_0 = \rho C_D U_r^2$

(4.15) $u_* = \left(\frac{\tau_0}{\rho}\right)^{\frac{1}{2}}$

(4.17) $u(z) = \left(\frac{u_*}{K}\right) \ln\left(\frac{z}{z_0}\right)$

From (4.15), we have $\tau_0 = \rho u_*^2$ (i)

From (4.17), $u_* = U_r \frac{K}{\left(\ln\frac{z}{z_0}\right)}$ (ii)

Substituting (ii) into (i): $\tau_0 = \rho \left[U_r \frac{K}{\ln\frac{z}{z_0}} \right]^2$

$$= \rho \left(\frac{K}{\ln\frac{z}{z_0}}\right)^2 U_r^2$$

$$= \rho C_D U_r^2$$

where $C_D = \left(\frac{K}{\ln\frac{z}{z_0}}\right)^2 = \frac{K^2}{\left(\ln\frac{z}{z_0}\right)^2}$

(3) If the surface atmosphere rises by 1°C, by how much will the longwave and sensible cooling increase?

$$LW = \sigma T_s^4 \doteq \sigma T_0^4 + 4\sigma T_0^3 (T_s - T_0)$$

$$\frac{\partial LW}{\partial T_s} = 4\sigma T_0^3 \longrightarrow \Delta LW = 4\sigma T_0^3 \cdot \Delta T_s$$

$$= 4 \times 5.67 \times 10^{-8} \times 288^3 \times 1$$

$$= 5.4 \ \mathrm{Wm^{-2}}$$

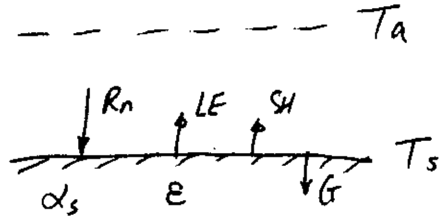
$$SH = C_p \rho C_D U (T_s - T_a)$$

$$\frac{\partial SH}{\partial T_s} = C_p \rho C_D U \longrightarrow \Delta SH = C_p \rho C_D U \Delta T_s$$

$$= \underset{\mathrm{Jkg^{-1}K^{-1}}}{1004} \times \underset{\mathrm{kgm^{-3}}}{1.2} \times 2 \times 10^{-3} \times \underset{\mathrm{ms^{-1}}}{5} \times \underset{\mathrm{K}}{1}$$

$$= 12 \ \mathrm{Wm^{-2}}$$

(4) The surface energy balance at the dry parking lot can be written as



$$R_n - LE - SH - G = 0 \qquad (i)$$

At the dry surface, $LE = 0$

and assume $G = 0$, then we can (i) as

$$R_n = SH \qquad (ii)$$

$$\because \quad R_n = S^{\downarrow}(1-\alpha_s) + \varepsilon(F^{\downarrow} - \sigma T_s^4) \qquad (iii)$$

$$SH = C_p \rho C_D U (T_s - T_a) \qquad (iv)$$

$$\therefore \quad S^{\downarrow}(1-\alpha) + \varepsilon(F^{\downarrow} - \sigma T_s^4) = C_p \rho C_D U (T_s - T_a) \qquad (v)$$

Substituting $\sigma T_s^4 \doteq \sigma T_a^4 + 4\sigma T_a^3 (T_s - T_a)$ into (v), and solving for $(T_s - T_a)$,

$$T_s - T_a = \frac{S^{\downarrow}(1-\alpha_s) + \varepsilon(F^{\downarrow} - \sigma T_a^4)}{C_p \rho C_D U + 4\varepsilon\sigma T_a^3} \qquad (vi)$$

Using $T_a = 27°C = 27 + 273.15 = 300.15\ K$

$S^{\downarrow} = 600\ Wm^{-2}$

$\varepsilon = 0.85$

$F^{\downarrow} = 300\ Wm^{-2}$

$\sigma = 5.67 \times 10^{-8}\ W m^{-2} K^{-4}$

$C_p = 1004 \times J\ Kg^{-1} K^{-1}$

$C_D = 2 \times 10^{-3}$

$U = 5\ ms^{-1}$

$\rho = 1.2\ Kg m^{-3}$

For asphalt, $\alpha_s = 0.1$, $T_s = 300.15 + \dfrac{600 \times (1-0.1) + 0.85 \times (300 - 5.67 \times 10^{-8} \times 300.15^4)}{1004 \times 1.2 \times 2 \times 10^{-3} \times 5 + 4 \times 0.85 \times 5.67 \times 10^{-8} \times 300.15^3}$

$= 300.15 + 23.5 = 323.65\ K = 50.5\ °C$

For concrete, $\alpha_s = 0.3$, $T_s = 300.15 + 16.5 = 316.65\ K = 43.5\ °C$

(5) If the parking lot is wet, the surface radiation balance (when G is neglected) is $\quad R_n = SH + LE \quad$ (i)

Since the air is saturated, following (4.34)

$$Be = \frac{SH}{LE} \quad (\equiv B_0)$$

$$\text{or} \quad LE = \frac{SH}{Be} \qquad \text{(ii)}$$

Substituting (ii) into (i): $\quad R_n = SH + \frac{SH}{Be} = SH\left(1 + \frac{1}{Be}\right) \quad$ (iii)

$$\therefore \; S^{\downarrow}(1-\alpha_s) + \varepsilon(F^{\downarrow} - \sigma T_s^4) = \left(1 + \frac{1}{Be}\right) C_p \rho C_D U (T_s - T_a) \qquad \text{(iv)}$$

using $\sigma T_s^4 \doteq \sigma T_a^4 + 4\sigma T_a^3 (T_s - T_a)$ in (iv),

$$\text{then} \quad T_s = T_a + \frac{S^{\downarrow}(1-\alpha_s) + \varepsilon(F^{\downarrow} - \sigma T_a^4)}{C_p \rho C_D U \left(1 + \frac{1}{Be}\right) + 4\varepsilon\sigma T_a^3} \qquad \text{(v)}$$

Be can either be determined by looking at Fig 4.10, or as follows.

$$\frac{1}{Be} = \frac{L}{C_p} \frac{\partial q^*}{\partial T} \qquad \text{(see 4.33)}$$

$$= \frac{L}{C_p} q^* \frac{L}{R_v T^2} \qquad \text{(see 4.35)}$$

$$= \frac{L}{C_p} \cdot 0.622 \frac{e^*}{p} \cdot \frac{L}{R_v T^2} \qquad \left(\text{Note that we have used } q^* = 0.622 \frac{e^*}{p}\right)$$

$$= \frac{L}{C_p} \cdot 0.622 \frac{1}{p} \cdot 6.11 \exp\left\{\frac{L}{R_v}\left(\frac{1}{273} - \frac{1}{T}\right)\right\} \cdot \frac{L}{R_v T^2} \qquad \text{(see Appendix B.3)}$$

$$= \frac{2.5\times10^6}{1004} \times 0.622 \times \frac{1}{1000} \times 6.11 \exp\left\{\frac{2.5\times10^6}{461}\left(\frac{1}{273} - \frac{1}{300.15}\right)\right\} \times \frac{2.5\times10^6}{461\times(300.15)^2}$$

$$= 3.435 \qquad (\text{or } Be \doteq 0.291)$$

$$\therefore \; T_s = 300.15 + \frac{600\times(1-0.1) + 0.85\times(300 - 5.67\times10^{-8}\times300.15^4)}{1004\times1.2\times2\times10^{-3}\times5\times(1+3.435) + 4\times0.85\times5.67\times10^{-8}\times(300.15)^3}$$

$$= 307.07\,K = 33.92\,^\circ C \qquad (\text{for asphalt})$$

If the surface is wet, the latent heat cooling reduces the surface temperature by $(50.5 - 33.92) = 16.58^\circ C$ for asphalt. If the air is not saturated, B_0, instead of Be should be used in (v). Since dry air promotes more evaporation, $B_0 < Be$. Then T_s will be lower.

(b) Comparing Fig 4.16 (c) and (d) reveals that Flagstaff has a large LE than Yuma, which implies Flagstaff is wetter and has more vegetation cover. Therefore, Flagstaff has a lower surface albedo (α_s) and a lower surface temperature (T_s).

$$\because \quad R_n = S^{\downarrow}(1-\alpha_s) + \varepsilon(F^{\downarrow} - \sigma T_s^4)$$

assuming both locations have about the same $S^{\downarrow}$, ε and $F^{\downarrow}$, then Flagstaff has a greater R_n than Yuma as the former absorbes more solar radiation and loses less longwave energy.