GEO387H: Approaches and Challenges in Ice Sheet Modelling

Toby Isaac

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Abstract

This is a review of some of the recent history of ice sheet modelling. A large source of uncertainty in future climate projections is the fate of the cryosphere: when, where, and how much ice will melt, and what effects that will have. The uncertainty is a function of the difficulty in modelling ice, both due to its complicated nature and due to our lack of information about the conditions within and beneath the ice sheets. This review will describe the rheology of ice, as well as the various boundary conditions that may be present at the edges of ice sheets. Next will follow a brief description of attempts to describe these interactions with models, beginning with one of the first tractable yet useful models, the shallow ice approximation (SIA). The two models that inform the IPCC's recent predictions about future melting are SIA models: these models, and experiments coupling them to global climate models (GCMs) will be discussed. Recent, more advanced models will be briefly described, and possible future directions of ice sheet modelling will be suggested.

1 Introduction

The ice sheets over Antarctica and Greenland play a critical role in our global climate. They contain $\sim$98% of the Earth's ice mass, and thus $\sim$80% of the Earth's freshwater, and with an albedo of 0.5 to 0.9, they reflect a large amount of incoming solar radiation [15]. The number with which most people are familiar, however, is the fact that the volume of water frozen in the ice sheets is equivalent to a $\sim$64 m rise in the sea level: $\sim$57 m from Antarctica and $\sim$7 m from Greenland [22]. The impact of a significant fraction of the ice sheets melting would be dramatic: recent estimates suggest 10% of the world's population lives at elevations below 10 m above sea level [24].

The catastrophic scenario that most alarms the public—that temperatures might rise enough to melt the ice sheets in this generation or the next's lifetime—is not considered likely by most climate scientists. Even in the most dire of the AR4 models, the ice sheet over Antarctica is stable, and increased insolation and temperature over Greenland would not produce the necessary energy to overcome the enthalpy of fusion for all of the ice for thousands of years [25]. Less dramatic changes in the ice sheets, however, could still significantly alter the Earth's climate systems. An example is given in the Global Climate Projections section of AR4 [25]: melting of ice in southern Greenland (the ice sheet in the most temperate climate) would dump more fresh water into the ocean where the Gulf stream feeds the Meridional Overturning Circulation, which would reduce its density, and could thus weaken or alter the overturning. Changes in ocean circulation would certainly bring long term changes to local climate systems, as these currents distribute the heat which drives weather patterns.

As policy makers increasingly turn to climate scientists for predictions about the effects of their decisions, so have climate scientists increasingly needed to clarify the sources of uncertainty in their predictions, and perhaps the greatest remaining source of uncertainty is the cryosphere. This uncertainty stems assuredly not from a lack of attention: the ice sheets have been cored, seismically imaged, monitored for precipitation and velocity by satellites, and changes in their mass balance have even been tracked through changes in the Earth's gravitational field [7]. Instead, there are two fundamental sources of this uncertainty.

The first is the nature of ice itself: it exhibits a wide variety of responses to different stresses. Whereas ocean dynamics can be modelled well by treating water as Newtonian and obeying the boussinesq approximation, and the atmosphere can be modelled well by treating air as obeying the ideal gas law, the viscous behavior of ice is highly non-linear, potentially anisotropic, and viscoelastic at brief time scales. Further complicating the material response of ice sheets is the presence of trapped air bubbles and minerals churned up from the bedrock.

The second is the inaccessibility of data: even with the ability to perfectly model the ice in any condition, our surface observations tell us little about the condition of the ice deep within the sheet. While ice cores allow us to test material properties, they have generally only been taken in the coldest sections of the ice, away from the complications of meltwater, which can collect at the bedrock and ease the friction between ice and sediment, facilitating basal sliding. Scientists have theorized that this basal sliding is the cause of the recent acceleration of the ice sheet in southern Greenland and western Antarctica, so predicting where meltwater accumulates, as well as where it may accumulate in the future, is critical to the success of a predictive model.

Stand-alone ice sheet models have been around for some time. The first model to gain widespread use was the shallow ice approximation [17], which reduces the problem of ice dynamics to a two-dimensional one, where the only important variable is the depth of the ice sheet. As computational power has increased, models have become more descriptive. The next generation of models allows for flow that varies with depth in the ice sheet, but in a layered fashion that is still a simplification over the full equations of motion. At the present, fully three-dimensional models are being developed, as well as models that incorporate some of the more complicated rheology described above.

Up until very recently, ice sheet dynamics were absent from all global climate models, and are now only included in a few. All of the full ice sheet models above are very computationally intensive, and require a resolution of details that is finer than exists for current global climate models. If these models can be verified to a certain degree, they can be used to produce simpler dynamics models which could be included with GCMs. Alternatively, GCMs and ice sheet models may be run in a coarsely coupled fashion: a GCM without ice sheet dynamics could simulate a decade of ocean and atmospheric circulation, which could then be treated as the forcing for the ice sheet model, whose condition at the end of that decade would then determine the extent of ice cover and the rate of fresh water outflow in the next decade of the GCM simulation.

Whatever the fashion that the inclusion of ice-sheet dynamics in GCMs takes, a process of verification should be established to see that the model agrees with observations. In particular, the combination of seismic imaging and ice-core dating gives a fairly accurate description of the sort of deformations that have happened in the past, which allows for a calibrating hind-cast to serve as a benchmark problem.

2 Challenges of Mathematical Description

(Most of the information for this section comes from [17], unless otherwise noted.)

2.1 Rheology

2.1.1 Polycrystalline Ice

Polycrystalline ice refers to ice that is made up of grains of ice crystals whose orientations are distributed randomly. This is the form of ice found in most ice sheets, since the ice is formed from snowflakes that land in a random fashion and then condense into ice under the weight of additional snow. Because of the random orientation of the grains, the ice can be considered isotropic.

When a stress applied to ice is weak (< 0.1 MPa) and instantaneous ($\mathcal{O}(1^{-15}$ sec)), the ice exhibits elastic behavior. While this behavior isn't relevant to modelling, it is certainly important, because without it we wouldn't have seismic imaging, which allows us to see through ice to the bedrock, and thus to know how deep the ice sheet is.

When a stress applied to ice is weak (< 0.1 MPa) and lasts for an appreciable amount of time, the ice exhibits viscoelastic behavior. Under constant stress, the ice passes through three phases of behavior: primary, secondary, and tertiary creep. During primary creep, the strain rate is large and decreasing, as if it will reach equilibrium like a typical solid. The ice then enters secondary creep, when the strain rate is constant, as if the ice is a typical fluid. Finally, once the stress has been applied long enough, the grains recrystallize in an anisotropic fashion that reduces its viscosity in the direction of the strain, and the strain rate approaches a final greater maximum. The times of the transitions and the various rates are all functions of the applied stress and the temperature of the ice, but the general form of the *creep function* appears as in figure 1.

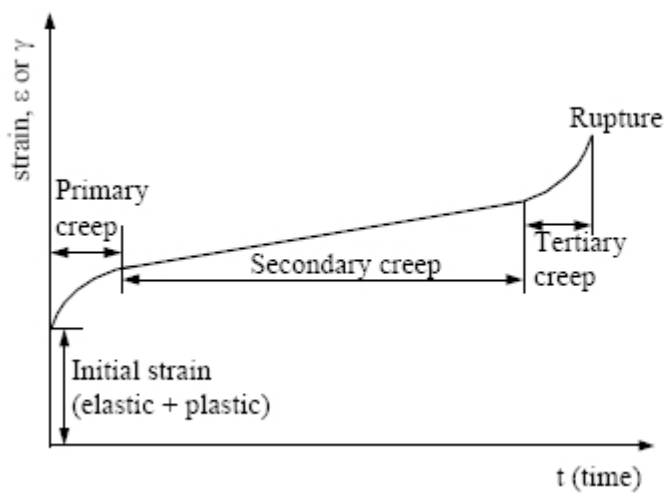

Figure 1: A typical creep curve

Finally, for large stresses (> 0.1 MPa), the elastic response of ice is negligible, and the ice behaves viscously. It does not, however, behave with linear or Newtonian viscosity. The relationship between stress and strain rate $\dot{\epsilon} = E(\sigma)$ that is most commonly seen is known as Glen's law,

$$\dot{\epsilon} = \text{sgn}(\sigma) A |\sigma|^n,$$

where n is a constant and A is dependent on temperature by an Arrhenius relationship, $A = \tilde{A}\exp(-Q/kT)$. The most common used value is $n = 3$, which fits well when $|\sigma| > 0.2$ MPa, although a value of $n = 2$, fits better for $|\sigma|$ between 0.1 and 0.2 MPa.

2.1.2 Single-Crystal Ice

When water freezes at the temperatures and pressures present on Earth, the ice that forms is in a hexagonal crystalline lattice known as ice Ih. Because the molecules are in their lowest energy state when they are in this lattice, they are most easily dislocated when the dislocation shifts the molecules from one lattice point to another. This means that, under applied stress, the molecules usually shift in three principle planes: the basal, prismatic, and pyramidal planes (see figure 2). Dislocation along the

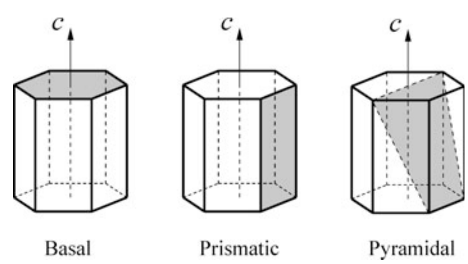

Figure 2: Planes of dislocation for ice Ih crystals (from [11]).

basal plane (whose normal is known as the *c-axis*) occur much faster then dislocations in the other two planes. This means that, for ice which is made up a single-crystal, shear strain rate in the basal plane occurs faster than other shear strains and much faster than tensile strains for equivalent stress.

As mentioned above in the discussion of creep, large sustained stresses can cause polycrystalline ice to recrystallize, which causes the previously uniform distribution of c-axes between the grains of ice to become biased. In particular, the combination of vertical pressure and horizontal shear stress causes more and more crystals to orient their c-axes vertically as the depth of the ice sheet increases. The response of the material then becomes more and more anisotropic with depth. Rheological analysis of one core sample from Greenland has shown that near the base of the ice sheet the effective viscosity of the ice to vertical compression is four times greater than that of polycrystalline ice, while the effective viscosity to horizontal shear was ten times less [10].

2.2 Boundary Conditions

2.2.1 Bedrock

In polar sections of the ice sheet, the temperature of the ice at the bedrock is below freezing, and in the absence of volcanic activity, the heat flux from the Earth is usually insufficient to melt the ice [23].

In these conditions, it is accepted that the ice sticks to the ground and has no velocity.

This is the most simple case, however. In temperate portions of the ice sheets, or in the presence of sufficient thermal heating, a thin layer of water exists between the ground and the ice that lubricates the flow. Additionally, as the ice presses against an obstacle, the increased pressure causes it to melt, which allows it to flow around the obstacle, at which point the low pressure trailing the obstacle causes the ice to refreeze: this process is known as *regelation*. The flow of the ice is still resisted by the roughness of the bedrock, which means that the shear stress of the flow is somehow dependent on this roughness and on the thickness and velocity gradient of the ice. The scale of the roughness is small compared to the width of the ice sheet, so it can be averaged out, and an effective bedrock slope can be determined: the problem is then to find a function $\mathbf{u}_b = B(\mathbf{T} \cdot \mathbf{n}_b)$ that gives the basal velocity of the ice at the bedrock in terms of the shear stress in the tangent plane. If the rheology of ice were linear, one might expect a boundary condition of the form

$$C\mathbf{T} \cdot \mathbf{n}_b = \mathbf{u}_b,$$

but because the rheology is non-linear, we instead have the *Weertman-type* boundary condition

$$C\mathbf{T} \cdot \mathbf{n}_b |\mathbf{T} \cdot \mathbf{n}_b|^{m-1} = \mathbf{u}_b,$$

where m is usually taken to be the same as n in Glen's law. The roughness of the ground beneath the ice sheet is expressed parametrically in the constant C, so a spatially varying value of C could potentially express the boundary condition for a wide variety of ground types.

When the ground is particularly soft, i.e. not composed of anchored stone, then the ice tends to pick up the sediment beneath it, creating a mixture known as *till*. Till can include rocks from pebble to boulder size and can include water within the space between them, which flows in a manner dependent on the temperature, water pressure, ice pressure, and pore size. The possible interactions are varied; for a more detailed description, see [8].

Another way to treat the boundary condition is the "stick-slip" approach outlined in [4]. There it is noted that where the ice thickness is on the order of 100 m (which for the major ice sheets is only near the margins), the relaxation time for ice can be on the order of hours to days, so that the short term behavior can be significantly effected by elastic effects. The paper uses a simple model of a threshold stress which must be overcome before the ice slides. Although the time scale that predictive models

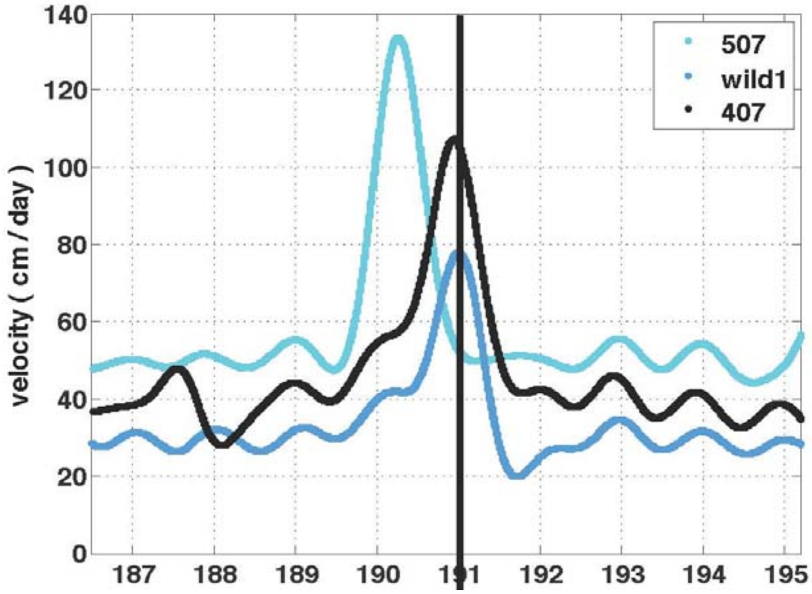

Figure 3: A time series of ice velocities, showing a brief jump (from [7]).

must run is much longer, and the ice sheets are much thicker than thin glaciers, the ice sheets are sometimes observed to make quick jumps, as shown in figure 3. If, for example, the base velocity is that predicted by a Weertman-type slip condition, and if such jumps in velocity happen at somewhat regular intervals throughout the year, then on the average the velocity of the ice would be underestimated. This suggests that some modification to the Weertman type slip condition may be necessary in predictive modelling.

It should also be noted that over the course of very long simulations, the bedrock cannot be considered static: as the weight of the ice redistributes, the rock underneath it deforms viscoelastically, and may rebound when the ice loses mass. For this reason, some of the models described later couple their ice sheets to a deformable lithosphere model.

2.2.2 Surface

There are two processes that affect the condition at the free surface of the ice sheets: accumulation and ablation.

The problem of accumulation is essentially an external input to ice sheet modelling: where, when, how much, and in what form precipitation falls. If it falls as snow, it becomes part of the mass of the ice sheet; if it falls as rain, it either freezes, accumulates on the surface, or seeps through cracks into the ice.

The problem of ablation is a good deal more complicated: when and where the surface melts is a question not only of the air temperature and wind, but also of the presence of water that has accumulated and the geometry.

The change in the shape of the surface can be described by a kinematic condition. Suppose we define a function $S(x,t)$ which is equal to the depth from the surface of a point x in the ice sheet: clearly we have $S(x,t) = 0$ when x is on the surface. If we track a particle X at a point $x(X,t)$ on the surface, and there is no accumulation or ablation, then that particle will always remain on the surface, meaning that $\partial/\partial t[S(x,t)] = 0$, or $\mathrm{d}S/\mathrm{d}t = 0$, where $\mathrm{d}\cdot/\mathrm{d}t$ is the material time derivative. If we know $a(x,t)$, the change in the surface height due to accumulation and ablation, then $\mathrm{d}S/\mathrm{d}t = a\cos\alpha$, where alpha is the slope of the surface at x.

2.2.3 Outflow

Ice sheets end at the sea in either an ice shelf or calving. It is important to understand the connection between the ice sheet and its outflow condition, because it has a large effect on the motion of the ice sheet. When an ice shelf is present, the shelf pushes against the ice sheet, resisting the pull of gravity; when there is calving, such resistance is almost non-existent. The modelling of ice shelves must take into account forcing from the ocean, and so presents its own unique problems. Some of the ice sheet models include their own ice shelf models, but for the purposes of this review, they are considered inputs to the ice sheet model. For an example of how the modelling of ice shelves is approached, see [29]. Calving involves the fracture of the ice, which occurs on time scales much too short to be modelled directly during predictive simulations; for a more detailed description of calving, see [6].

3 Models

3.1 The Shallow Ice Approximation

3.1.1 Equations

All continuum models begin with the basic equations of motion:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0;$$
$$\rho\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\nabla p + \nabla\cdot\mathbf{T} + \rho\mathbf{g},$$

where ρ is density, $\mathbf{v}$ is velocity, p is pressure, $\mathbf{T}$ is the deviatoric stress tensor, and $\mathbf{g}$ represents body forces (only gravity in ice ice sheet modelling). Densification does take place during the transition from snow to firn to ice, but the layer of snow and firn is thin compared to the depth of the ice sheet, and ice exhibits very little density change with temperature, so one more often sees $\nabla \cdot \mathbf{v} = 0$. Additionally, the slow nature of the ice sheet flow means that the acceleration terms in the momentum equations are negligible, giving us the Stokes approximation, $\nabla p = \nabla \cdot \mathbf{T} + \rho \mathbf{g}$. The lack of time in the Stokes approximation might lead one to think that it cannot by used to simulate dynamics, but in fact the velocity field drives the free-surface and energy evolution equations. Since the material response present in $\mathbf{T}$ changes with temperature and the pressure field must match atmospheric pressure at the free surface, a different Stokes problem must be solved for each moment in time.

The shallow ice approximation (SIA) is based on three main assumptions. The first is that the expanse of the ice sheet is much greater than its thickness, and that the bedrock and ice-surface slopes are small. This is clearly a good assumption, as the ice sheets are on the order of 1-3 km thick and the bedrock varies at most on the order of 1 km in the vertical direction, whereas they cover millions of square kilometers.

The second assumption is that the flow is essentially two dimensional, $w \ll u, v$. In most cases, this is a good assumption: As the Vostok ice core shows, it can take ~400,000 years for the ice to reach the bottom of the ice sheet ($\sim$ 3 km), whereas the surface velocity of portions of the ice sheet are on the order of 1km/year or 1km/10years. Mathematically, this means that we can assume $\dot{\epsilon}_{zz} = t_{zz} = 0$.

The third assumption is that the change of stress and strain in the vertical direction is much greater than in the horizontal direction. This is also a good assumption in most cases: we expect the velocity of two points at the same depth, but a kilometer distant, to be more similar than the velocities of two points at the same map location, but separated by a kilometer in depth, due to the traction of the bedrock. Mathematically, this means that we can assume $\partial t_{(\cdot)}/\partial x = \partial t_{(\cdot)}/\partial y = 0$ and that $\dot{\epsilon}_{xz} = \frac{1}{2}\partial u/\partial z$ and $\dot{\epsilon}_{yz} = \frac{1}{2}\partial v/\partial z$. This leaves us with the following set of equations:

$$
\begin{aligned}
\frac{\partial w}{\partial z} &= -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right); \\
\frac{\partial p}{\partial x} &= \frac{\partial t_{xz}}{\partial z}; \\
\frac{\partial p}{\partial y} &= \frac{\partial t_{yz}}{\partial z}; \\
\frac{\partial p}{\partial z} &= \rho g.
\end{aligned}
$$

The fact that the z-component of the momentum equation has reduced to the hydrostatic equation is a strong hint that we can depth average these equations to reduce our number of variables and our number of dimensions: in fact we can reduce the system to H, the height of the surface of the ice over the bedrock, which is a single variable in two dimensions. For a detailed derivation of the equations governing H, see [17]. it suffices here to say that when the depth averaged equations of motions are combined with the surface and bedrock boundary conditions and the surface kinematic condition, we get an equation of the form

$$\frac{\partial H}{\partial t} = a + \nabla \cdot [F(\nabla H, \nabla b; E, B)] + G(\nabla b; E, B),$$

Where a is the accumulation function, ∇b is the slope of the bedrock, E is our function that relates stress to strain rate, and B is our function that relates basal stress to basal sliding velocity.

3.1.2 SIA Models: GISM and SICOPOLIS

The reduction in the number of variables and dimensions makes the SIA computationally cheaper and easier to program than the full Stokes equations, so until recently the only large scale models that were able to incorporate atmospheric, oceanic, and thermal forcing used the SIA as their basic equations.

The section of the AR4 that describes climate projections, [25], cites three studies that attempt to model the dynamics of ice in the coming century: Gregory et al. 2004 [12], Huybrechts et al. 2004 [19], and Ridley et al. 2005 [27]. Gregory and Huybrechts are co-authors in all three papers.

The first article, Gregory et al. 2004, does not consider ice dynamics explicitly, but instead cites an older study [20], where a simple accumulation and ablation model predicted that if the mean temperature over Greenland were to rise 2.7°C, then ablation would dominate accumulation in all circumstances, and the ice would eventually melt. With this number, the authors then ran several atmosphere-ocean general circulation model (AOGCM) simulations where the CO_2 concentration was raised, and the mean temperature was allowed to come to equilibrium. Several model parameters and the CO_2 concentration were varied to reflect uncertainty in the modelling. In almost all simulations, the temperature passed the 2.7°C threshold, meaning that melting would be present over the whole ice sheet. This type of study does not predict how quickly the ice will melt, however. The article does cite two papers that provide numerical predictions of the rate of melting: Huybrechts and de Wolde 1999 [18], which describes the Greenland Ice Sheet Model (GISM), and Greve 2000 [14], which runs simulations using the SICOPOLIS model. (For a technical description of SICOPOLIS, see [13].)

Besides being SIA models, there is a lot of similarity between the two models. Both use a power of $n = 3$ for Glen's law, and both use Weertman-type basal sliding mechanisms with $m = n$. They both allow for temperature dependence of the material response, and each ice sheet is coupled to a thermo-viscoelastic lithosphere model for the bedrock. An attractive feature of both models is that they model the energy equation in a three dimensional fashion: at each time step, once the height has been calculated, the full three dimensional velocities within the ice can be computed and used for the advection term in the energy equation; the updated temperature field then affects the depth averaging that is used to compute the evolution of the height.

There are ways in which the models differ. GISM handles the behavior of meltwater in a parametric way, where it could refreeze of runoff as a function of the atmospheric forcing. SICOPOLIS allows for the temperate ice to have a meltwater content permeate the ice, and tracks its concentration with and advection diffusion equation. The GISM also allows for a difference in rheology between the higher, newer ice and the deeper, older ice by the inclusion of an enhancement factor in the stress-strain rate relationship that can be tuned. The GISM also has a full ice-shelf model, whereas SICOPOLIS treats marine ice in a less advanced, parametric fashion. Numerically, both models are finite difference based, but the GISM uses fully second-order derivative approximations and fully implicit time stepping, whereas SICOPOLIS uses 2nd order diffusion and 1st order advection operators and a semi-implicit/explicit time stepping formula.

First we consider the experiment run by Huybrechts and de Wolde. The authors did not find the precipitation predicted by AOGCMs at the time to be satisfactory for the length scales used in the study, so precipitation and temperature forcing came from a zonal mean energy balance model (see appendix D of [19] for more details). The temperatures were allowed to fluctuate around a daily mean that is forced by the CO_2 concentration, and precipitation was based on random fluctuations around current averages. As a verification technique, the models were started in the year 1990, and the parameters were tuned so that the prediction for the period 1990-2003 best matched reality. These parameters were then used for the rest of the experiment. Several different emissions scenarios were chosen to match the radiative forcing in the three IPCC IS92 scenarios (two-fold, four-fold, and eight-fold increase in CO_2 concentration). The results of the experiments can be seen in figure 4: although Greenland's mass loss was predicted to contribute to sea level rise, the net change when Antarctica was included spanned -6 to $+25$ cm sea level rise by the year 2100, with the middle emissions scenario predicting a slight decrease in sea level. The model also predicted a ∼2-3% decrease in the surface area and volume

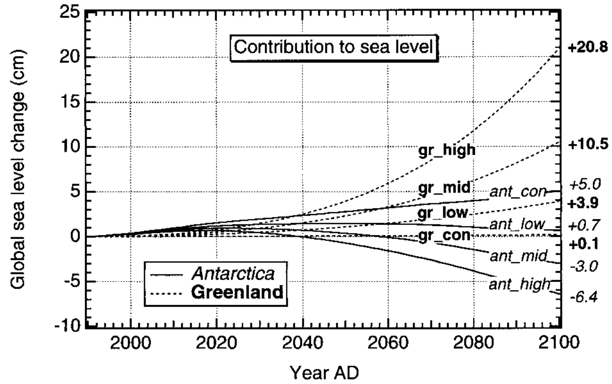

Figure 4: Ice loss in sea level rise equivalent for Greenland (dotted lines) and Antarctica (solid lines) under low, medium, and high CO_2 concentration increases (from [18]).

of Greenland by the year 2130: these values were $10 - 20\%$ less than the shrinkage that equivalent simulations without ice dynamics predicted. The experiment also found that simulations without ice dynamics overestimated the increase in ice volume of Antarctica in comparison to the simulations with ice dynamics.

Greve's experiments were less concerned with the comparison of dynamic to static models and more concerned with the sensitivity of the SICOPOLIS model to its parameters, and with the possible existence of a "critical threshold" of temperature or CO_2 that would cause a dramatic increase in the loss of ice mass. The atmospheric forcing is somewhat simpler, with a the mean temperatures initialized at 1991 mean temperatures and allowed to asymptotically approach a predetermined temperature increase (12 runs were performed with ΔT varying from $+1^\circ$C to $+12^\circ$C), snowfall is kept constant. After a simulation time of 1000 years, the simulation predicted a wide range of the remaining ice volume in Greenland, from ∼90% for a 3°C increase, to ∼10% for a 10°C increase (see figure 5). The changes

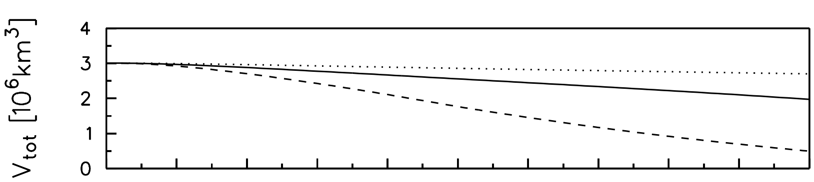

Figure 5: Ice mass remaining over 1000 year period under 3°C, 6°C, and 10°C temperature increases (from [14]).

were not consistent with a "critical threshold" however: the volume remaining varies smoothly with the temperature increase. Qualitatively, the remaining ice structure for Greve's 10°C increase simulation looks very similar to Huybrechts's and de Wolde's simulation for an eight-fold increase in CO_2

concentration (see figure 6).

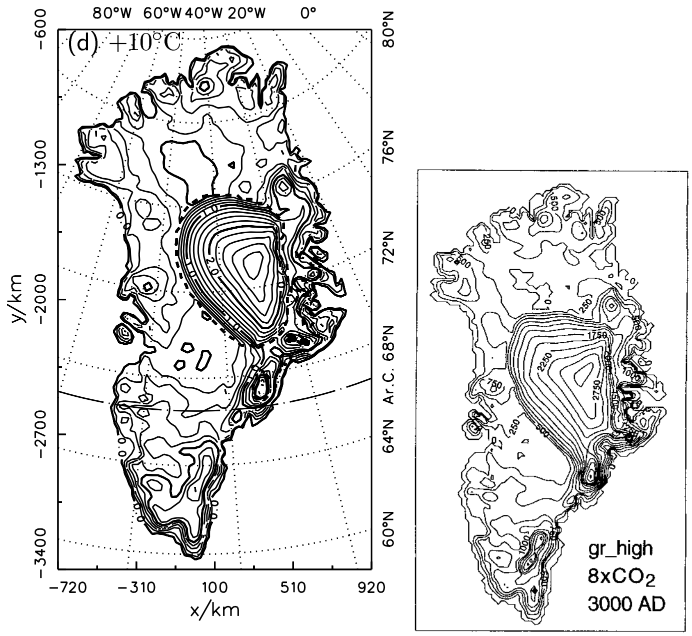

Figure 6: Remaining ice mass after 1000 years in [14] and [18].

The second paper cited by the AR4, Huybrechts et al. 2004, is a set of experiments with an updated version of the GISM and coupling with atmospheric and ocean forcing from time slice data to see if the inclusion of changing atmospheric and oceanic circulation patterns effect the mass balance results from their earlier paper. Time slice data involves the running of a coarse grid coupled AOGCM, whose sea surface temperature is then used as forcing for a higher resolution atmosphere-only GCM. The authors compared the results from two different time-slice experiments: one where the coarse and fine models come from the ECHAM4 GCM and the OPYC3 ocean model (see [2]), and the other where the coarse and find models are the Hadley Centre models HadCM3 and HadAM3H, respectively. The experiments were run with the middle forcing from the three IPCC IS92 scenarios that were used in the previous experiment. There was fairly good agreement between the two time-slice studies: and each suggested that the trends of decreasing mass balance in Greenland and increasing mass balance in Antarctica from the 1999 study were correct, but revised the amplitudes of these changes downward. The net effect of the predictions for the year 2100 was a -0.4cm change in sea level from the baseline, but with error significant enough to make this indistinguishable from null change.

The final paper cited by the AR4, Ridley at al. 2005, uses true two-way coupling between the GISM and the HadCM3 GCM to see if the change in the ice mass of Greenland has a significant effect on climate beyond the increase in sea level. The authors first ran an experiment from the GISM-HadCM3 coupling with pre-industrial CO_2 levels to see if the equilibrium ice geometry reflected current geometry, and then increased the CO_2 to four times the pre-industrial level and ran the experiment for 3000 years (with the GCM updating less frequently after 350 years for computational reasons). The results of the predictive experiment began similarly to those without two-way coupling, with the ice thinning and then disappearing in southern and western Greenland. Once this occurred, though, the deglaciated land area heated differentially to the remaining ice sheet, which set up a convection cell (see 7), bringing frigid air to the ablation zone and slowing the rate of melting. The experiment also found that the two-way

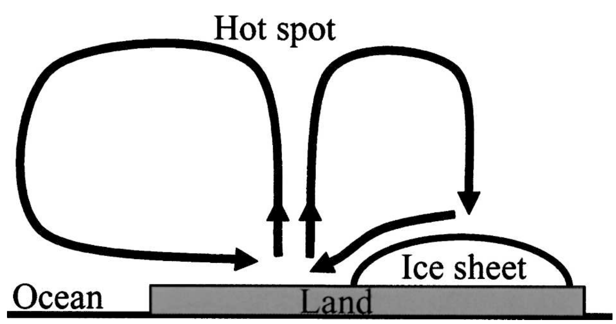

Figure 7: Convection cell that develops after partial ice sheet melting (from [27]).

coupling induced some additional weakening of the thermohaline current, but not enough to alter ocean circulation patterns, which is one of the scenarios discussed in AR4.

3.2 Longitudinal Improvements to the Shallow Ice Approximation

The SIA makes assumptions that the horizontal changes in the ice sheet are gradual, so naturally one would expect that areas where this does not hold, such as in the presence of steep bedrock topology, would be sources of error in SIA models. Longitudinal improvement schemes gradually reintroduce components of stress, or approximations thereof, back into the SIA so that the equations more closely resemble the full Stokes equations. A full and insightful analysis of these schemes is given by Hindmarsh in [16], which shows that term must be added with care: some proposed improvements actually impair the ability of the model to capture high frequency dynamics. A consequence of adding these additional terms back in is that the u and v components of velocity are reintroduced as field variables (i.e. they cannot be determined solely from the height), and in some longitudinal schemes the equations again become three dimensional. These schemes still have an advantage over full Stokes in that they involve two field variables as opposed to four (w and p are not included), and in that they are better conditioned systems of equations. Hindmarsh points out, though, that all approximations based on SIA grow poorer as the proportion of basal sliding to the surface velocity of the ice increases. This is unfortunate, as the next challenge for numerical ice sheet modelling is to attempt to capture the sort of lubricated basal sliding that is occurring in southern Greenland an western Antarctica.

3.3 Full Stokes

The time seems to be ripe for full Stokes modelling of ice sheets. There are several reasons for this. The first is that increased computing power means that more and more scientists have the brute force to tackle the full system of equations. The second reason is that pointed out above: the regimes where the SIA breaks down, such as quick basal sliding and steep gradients, are often the regimes that most interest scientists. Finally, flexible physics packages that handle the discretization of partial differential equations are allowing climate scientists to run three dimensional experiments without have to concern themselves too much with numerical or coding issues. A few new models that reflect this trend are given by Johnson and Staiger in [21] and by Durand et al. in [9], who use the packages COMSOL [1] and Elmer [3], respectively. In figure 8 taken from Johnson and Staiger's paper, even though the actual bed gradients in the Ferrar glacier are not actually as steep as the vertically stretched image makes them appear, they are still significant, and the large variation of the ice thickness also would make SIA a poor fit.

An invaluable resource is the large collaboration by Pattyn et al. [26] that surveys the recent full Stokes and longitudinal improvement models, categorizes them by the type of equations that they use to model, and establishes six benchmark problems with which to test them. In the tests for which an analytical solution was known, the full Stokes solutions were generally on the mark, while there was a wider spread in the longitudinal improvement models.

3.4 Future Directions

Consider the following items:

- As demonstrated in figure 9 of the basal stresses in one of the bench mark problems in Pattyn et al., the basal stresses are in some places below the 0.1 MPa threshold where ice should be considered viscoelastic.

- Figure 10 taken from [7], which shows the estimated tensile stress experienced by the ice sheet. After the ice experiences enough stress to cause fractures, meltwater seeps more easily through it

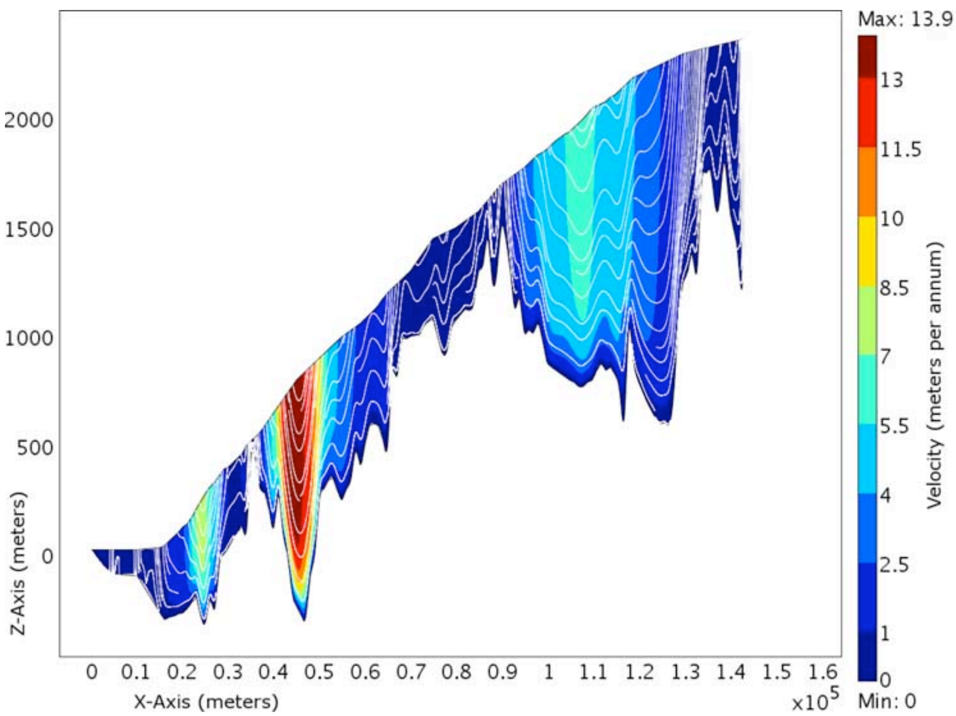

Figure 8: Diagram of the ice depth in the Ferrar glacier (from [3]).

to the base and lubricates sliding.

- Seddik et al. [28] have developed a model to describe how stress over time affects the distribution of c-axis orientations within polycrystalline ice.

These items all point towards stress-history dependence as being the next important development in ice sheet modelling. Incorporating history dependence for things like stress tensor fields is not trivial, in particular because a Lagrangian description is best suited to describe history dependent properties, which is at odds with the Eulerian description best suited to Stokes flow.

On the other hand, the resolution required for ice sheet models is already finer than the resolution for GCMs, and adding more bells and whistles may only delay the time when GCMs may be run fully coupled with models that incorporate ice sheet dynamics. An alternative avenue of development is to find reduced models of the existing ice sheet models that still capture the relevant dynamics in the most commonly occurring circumstances.

For projections to be trusted, there must be some sort of proof of a models validity. It is tempting as a modeller to be content with a model that reproduces one or two statistics for which there exists empirical data, even if the importance of that statistic to the intended use of the model is debatable.

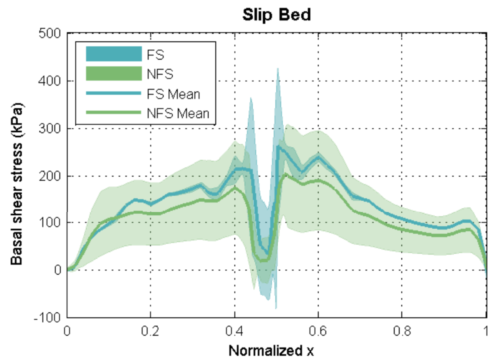

Figure 9: Basal stresses from benchmark experiment in [26].

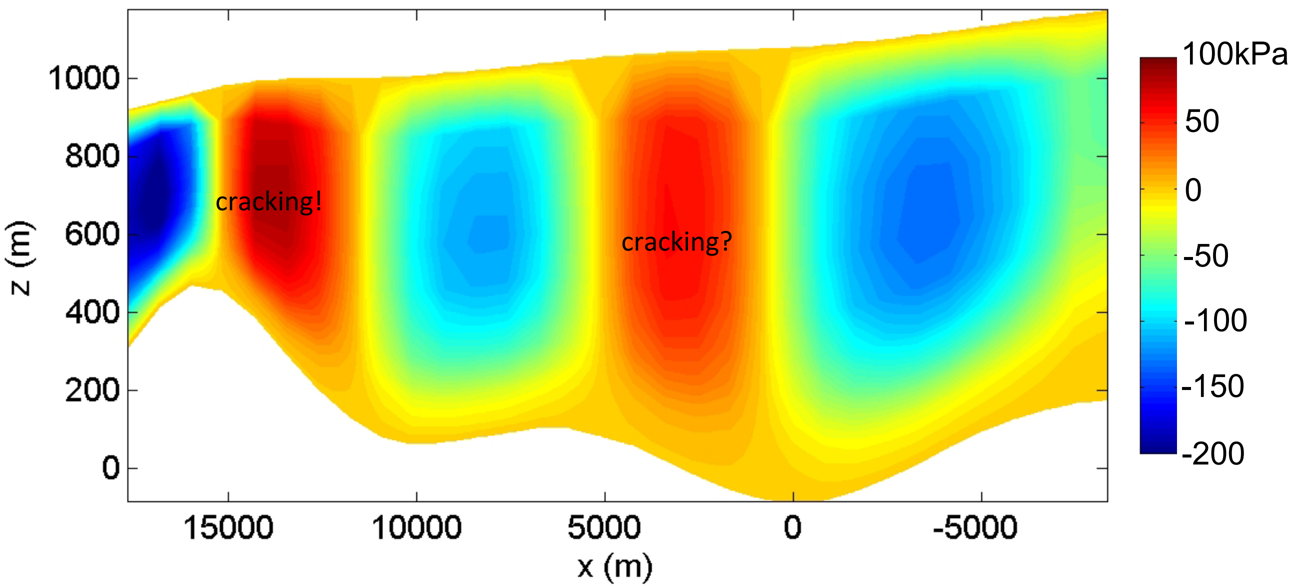

Figure 10: Tensile stresses from the Greenland ice sheet, with those stresses exceeding the fracture strength of ice labelled (from [7]).

For ice dynamics, the clear best test of a model is whether it models past dynamics for known forcing. We have not been observing the dynamics of ice sheets for long, however, so the choice of data is not so obvious. Baldwin et al. [5] point out that our ability to detect layers in the ice through seismic imaging allows us to track isochronous surfaces through the ice, and the air and particulates within the layers above an isochronous surface also encode the sort of atmospheric forcing that the surface experienced as it descended. Thus a test experiment for an ice sheet model is defined: an ice sheet is initialized from a known or guessed configuration, and its internal velocity fields are recorded as its initial surface is buried under the surface. After the velocity fields are constructed, tracer particles that begin at the surface will show where the initial surface now sits. This surface can be compared to the isochrones in the ice sheet to validate the model.

4 Conclusion

In this review, we covered the difficulties in modelling ice dynamics that are responsible for the uncertainty surrounding the nature and future of the cryosphere. The primary difficulty stems from ice as a material, because it can exhibit nonlinear, history dependent, and anisotropic responses to stress. We reviewed the various proposed basal boundary conditions, as well as surface and outflow conditions. There followed a brief description of the core model that first attempted to capture these aspects, the shallow ice approximation. The estimates of future melting used by the IPCC AR4 were based on two SIA models, the GISM and SICOPOLIS, so these two models were reviewed in detail. The next important study provided forcing for the GISM from an AOGCM, which revised down the rate of melting, and then a coupled AOGCM-GISM model revised it down further and showed changes in local weather patterns. The next generation of models, whether longitudinal improvement schemes or full stokes models, are better suited at capturing the dynamics in the situations about which uncertainty is the highest, such as quick basal sliding and steep slopes, and a set of benchmark problems has been developed to test these new models. Future directions for ice sheet modelling include stress-history dependence, better coupling with GCMs, and more rigorous validation.

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