Approaches and Challenges in Ice Sheet Modelling

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The Equations of Motion

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \qquad \text{(Continuity);}$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \nabla \mathbf{v} \cdot \mathbf{v} \right) = -\nabla p + \rho \mathbf{g} + \nabla \cdot \mathbf{T} \qquad \text{(Momentum);}$$

$$\mathbf{T} = F(\dot{\mathbf{E}}; T, \dots) \qquad \text{(Stress-Strain);}$$

$$\dot{\mathbf{E}} = \nabla \mathbf{v} + \nabla \mathbf{v}^{\mathrm{T}}.$$

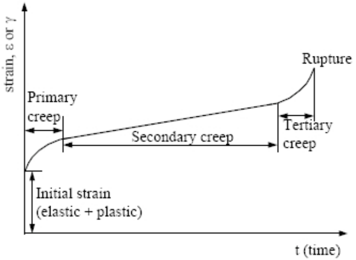
The Stokes Approximation

$$
\begin{aligned}
\nabla \cdot \mathbf{v} &= 0 && \text{(Continuity);} \\
\nabla p &= \rho \mathbf{g} + \nabla \cdot \mathbf{T} && \text{(Momentum);} \\
\mathbf{T} &= F(\dot{\mathbf{E}}; T, \ldots) && \text{(Stress-Strain);} \\
\dot{\mathbf{E}} &= \nabla \mathbf{v} + \nabla \mathbf{v}^{\mathrm{T}}.
\end{aligned}
$$

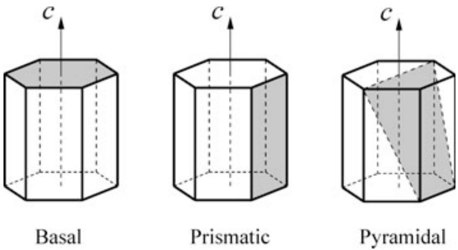
- Good for ice at glacial time scales and pressures
- How is this time dependent?
- Must define F and boundary conditions

Rheology: defining F^{-1}

1. High Stress
   - Viscous, but nonlinear
   - **Glen's power law**: $\dot{\varepsilon} = A\sigma^3$
   - A increases with increasing temperature
2. Low Stress
   - Viscoelastic
   - *Creep* :



Anisotropy

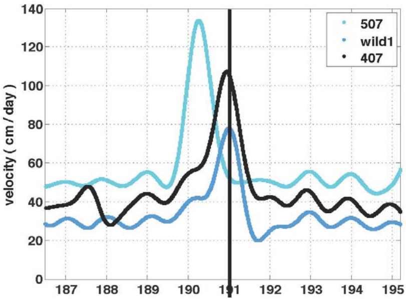


(Faria, 2003)

- Shear easiest in basal plane
- Polycrystalline ice becomes organized with time, pressure, and shear
- Order of magnitude difference in effective viscosity

Basal boundary conditions

1. Cold boundary
   - No slip: not complicated
2. “Warm” boundary
   - Thin layer of water lubricates
   - **Weertman slip**: $v = C\sigma^3$
   - *C* is a parametric roughness constant: how to determine?
   - Is “Stick-Slip” more appropriate?



(Catania, 2008)

Shallow Ice Approximation (SIA)

$$
\begin{aligned}
\partial_x \sigma_{xx} + \partial_y \sigma_{xy} + \partial_z \sigma_{xz} &= -\partial_x p; \\
\partial_x \sigma_{xy} + \partial_y \sigma_{yy} + \partial_z \sigma_{yz} &= -\partial_y p; \\
\partial_x \sigma_{xz} + \partial_y \sigma_{yz} + \partial_z \sigma_{zz} &= -\partial_z p - \rho g; \\
\sigma_{ij} &= F(\dot{\varepsilon}_{ij}; T, \dots); \\
\dot{\varepsilon}_{ij} &= \partial_j v_i + \partial_i v_j.
\end{aligned}
$$

Shallow Ice Approximation (SIA)

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\begin{aligned}
\partial_z \sigma_{xz} &= -\partial_x p; \\
\partial_z \sigma_{yz} &= -\partial_y p; \\
0 &= -\partial_z p - \rho g; \\
\sigma_{ij} &= F(\dot{\varepsilon}_{ij}; T, \ldots); \\
\dot{\varepsilon}_{iz} &= \partial_z v_i.
\end{aligned}
$$

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\partial_z \sigma_{xz} &= -\partial_x p; \\
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\sigma_{ij} &= F(\dot{\varepsilon}_{ij}; T, \dots); \\
\dot{\varepsilon}_{iz} &= \partial_z v_i.
\end{aligned}$$

Hydrostatic approximation $\Rightarrow$ depth average!

Shallow Ice Approximation (SIA)

$$\partial_t h = \nabla \cdot [G(\nabla h)] + B$$

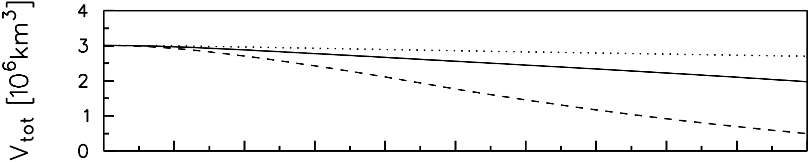
G and B are dependent on rheology F, boundary conditions, and basal slope.

Prediction with SIA

AR4 draws on four studies to predict changes in ice mass:

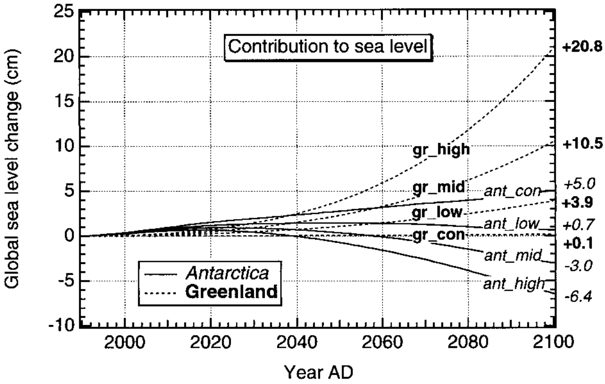
1. Huybrechts and de Wolde 1999 (GISM/AISM model + simple atmosphere)
2. Greve 2000 (SICOPOLIS model + simple atmosphere)
3. Huybrechts et al. 2004 (GISM/AISM model forced by GCM)
4. Ridley et al. 2005 (GISM/AISM coupled with GCM)

Greve 2000 (SICOPOLIS)



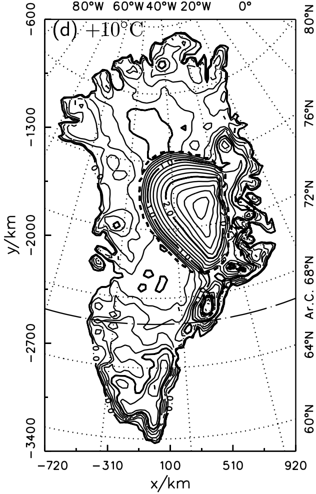
3, 6, and 10 degree increase in mean temperature over 1000 years (Greenland only).

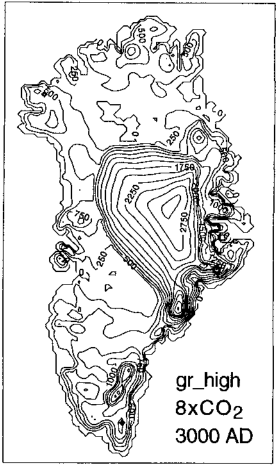
Huybrechts and de Wolde 1999 (GISM/AISM)



Three different CO2 scenarios for next 100 years: wide spread of outcomes

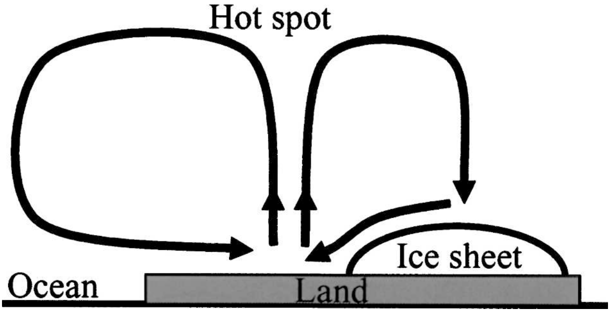
GISM and SICOPOLIS comparison





Remaining ice mass after 1000 years

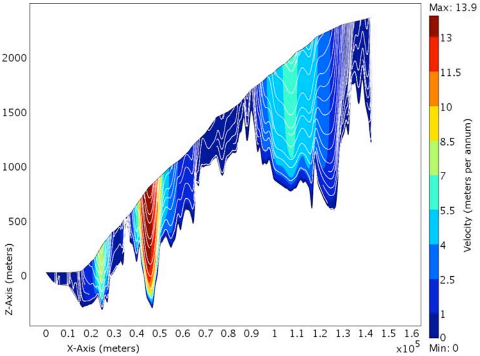
Ridley 2005: Coupling



A new convection cycle develops, slows melting.

Newer stand-alone models

1. Longitudinal improvements: remove fewer terms
2. Full Stokes where SIA is not appropriate



(Johnson and Staiger, 2007)

Future Directions

- Verification (Pattyn, 2008)
- Stress history dependence
- Rigorous model reduction techniques